Applied Mathematical Modelling 39 (2015) 1119–1136

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

# Model trees and sequential minimal optimization based support vector machine models for estimating minimum surface roughness value

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ARTICLE INFO

*Article history:*
Received 3 September 2012
Received in revised form 6 June 2013
Accepted 22 July 2014
Available online 8 August 2014

*Keywords:*
Surface roughness
Model trees
SVM
End milling

ABSTRACT

Average surface roughness value ($R_a$) is an important measure of the quality of a machined work piece. Lower the $R_a$ value, the higher is the work piece quality and vice versa. It is therefore desirable to develop mathematical models that can predict the minimal $R_a$ value and the associated machining conditions that can lead to this value. In this paper, real experimental data from an end milling process is used to develop models for predicating minimum $R_a$ value. Two techniques, model tree and sequential minimal optimization based support vector machine, which have not been used before to model surface roughness, were applied to the training data to build prediction models. The developed models were then applied to the test data to determine minimum $R_a$ value. Results indicate that both techniques reduced the minimum $R_a$ value of experimental data by 4.2% and 2.1% respectively. Model trees are found to be better than other approaches in predicting minimum $R_a$ value.



## 1. Introduction

Machining is the process of removing material from a work piece to transform it to a desired shape. Different machining processes include conventional processes such as grinding, milling, drilling etc. and non-conventional processes such as electrodischarge machining, electrochemical machining, waterjet cutting etc. The milling process uses a rotating cutter for material removal. Two main types of milling are: peripheral milling and face milling. In peripheral milling the machined surface is obtained parallel to the spindle rotation whereas in face milling the machined surface is produced normal to the spindle rotation. Further the milling process is classified as conventional milling and climb milling. In conventional milling the direction of feed of workpiece is against the cutter rotation whereas in climb milling, both the cutter rotation and the workpiece feed are in the same direction. End milling involves a mix of peripheral and face milling and employs bottom and edges of the milling cutter. Fig. 1 shows the geometry of milling process. The important parameters are cutting speed, feed rate, depth of cut and rake angle. Cutting speed is the speed at which the tool tooth cuts through the workpiece. It is expressed in meters per minute or surface feet per minute (SFPM). Feed rate is the speed at which the workpiece is fed into the cutting tool and is expressed in inches per minute or millimetre per minute. Depth of cut specifies the penetration of the milling

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http://dx.doi.org/10.1016/j.apm.2014.07.026
0307-904X/

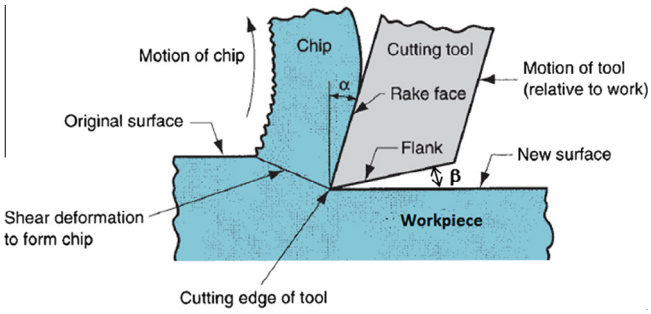


**Fig. 1.** Geometry of milling process showing the workpiece, cutting tool, newly formed surface, chip and different angles (adapted from [1]. $\alpha$ is the rake angle which specifies the direction of chip flow. The angle $\beta$ between flank and the new surface is called the clearance angle.

cutter into the workpiece and indicates the amount of material removed in each pass. The rake angle specifies the direction of chip flow as the workpiece is machined.

One of the commonly used measures of the performance of any machining process, such as milling is the surface roughness of the machined work piece. Surface roughness is defined as the vertical deviations of the surface from its ideal form. Different surface roughness parameters are in use such as root mean square roughness ($R_q$), roughness average ($R_a$) and maximum peak-to-valley roughness ($R_y$ or $R_{\max}$), etc. [2]. However, roughness average ($R_a$) is the most widely used parameter. It is defined as the integral of the absolute value of roughness profile height over the evaluation length i.e.

$$R_a = \frac{1}{l}\int_0^l |y(x)|d(x),$$

where $l$ is the sampling length and $y$ is the ordinate of the profile curve as shown in Fig. 2.

A low surface roughness value ($R_a$) indicates high machining performance and a better quality work piece and vice versa. A high quality work piece with lower surface roughness will be more corrosion resistant and have high creep life and fatigue strength. Therefore, considerable research effort has been put into the development of models that could predict machining parameters resulting in minimum $R_a$ value. These models establish relationship between cutting conditions (usually expressed in terms of cutting speed, feed rate, depth of cut and rake angle) and the resulting $R_a$ value. Artificial Neural Networks (ANN) have been used in the prediction of surface roughness in drilling [3], abrasive waterjet machining [4], CNC lathe [5] and end milling [6,7]. Least square support vector regression (LS-SVR) is employed in [8,9] to predict surface roughness in the end milling process. In [7] regression and ANN models for predicting minimum $R_a$ value in the end milling process were developed. As compared to experimental data, they achieved a reduction in minimum value of $R_a$ by 1.57% and 1.07% by regression and ANN models respectively. References [10,11] provide a detail review of techniques for predicting surface roughness in different machining processes. In this paper we use sequential minimal optimization (SMO) based support vector machine (SVM) [12,13] and model trees [14,15] to determine minimum $R_a$ value in the end milling process.

### 1.1. Contributions

Our main contributions in this paper are as follows:

1. The techniques of model trees and sequential minimal optimization based support vector machines (SMO-SVM) have not been used before to model surface roughness. Our work builds upon and is an extension of the work previously reported in [7]. In [7] the authors modeled surface roughness data using simple regression and Artificial Neural Network (ANN). We have modeled, for the first time, the same data using model trees and SMO-SVM. Our contribution is that we have shown that both of these techniques can be used to accurately model real experimental data of an end milling process and predict the cutting conditions that can result in minimal surface roughness value, thus ensuing increased quality of the machined work piece.

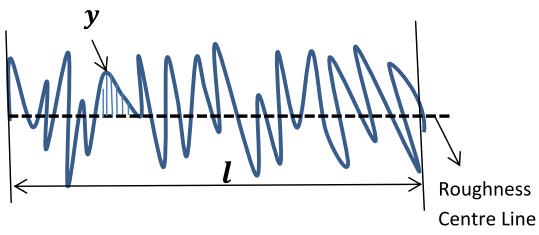


**Fig. 2.** Definition of surface roughness. $l$ is the sampling length and $y$ is the ordinate of the roughness profile curve.

2. We have described the way to select appropriate data for training and testing. An experimental dataset is divided into two different types of training and test datasets. An explanation of the composition of these two different types of training and test datasets and their impact on the accuracy of the regression models developed using SMO-SVM and model trees is provided. We have highlighted which composition is better out of the two and the causes of improvement in the accuracy of the developed regression models due to it.
3. We have in this paper, shown improved results than the previously reported ones [7] for the problem of finding minimum surface roughness value $R_a$ for three type of cutting tools: uncoated, TiA1N coated and SNTR coated. We have shown that the techniques of model trees and SMO-SVM combined with *carefully crafted training and test sets* perform better as they can identify surface roughness values lower than one obtained through simple regression and ANN. The validity of the results is carried out through graphical plots and tests for statistical significance.

This paper is organized as follows: Section 1 introduces the significance of finding minimum value of surface roughness and describe some existing techniques, Section 2 discusses the techniques of model tree and SMO-SVM, Section 3 describes the data set used in this paper and elaborates the results of employing model trees and SMO-SVM for estimating minimum $R_a$ value, Section 4 evaluates the results using $t$ tests and making comparison with [7], finally Section 5 concludes the paper.

## 2. Models for predicting surface roughness

Developing mathematical models for predicting $R_a$ value is a challenging task because surface roughness value is the result of complex interdependence between various process parameters. In this section we describe the techniques of model trees and SMO-SVM that can be employed to generate reliable regression based models for $R_a$ value prediction. To the best of our knowledge, this is the first work that employs these two techniques for predicting minimum $R_a$ value. These two techniques are selected as they belong to two different families of algorithms. The models produced by SVM are opaque like the models produced by ANN. Model trees, on the other hand, belong to decision tree family. As compared to ANNs, they produce more understandable models by providing greater visibility into the relationship between variables that build the model.

### 2.1. M5 model trees

M5 model trees [14,15] have multivariate linear regression models at their leaf nodes, differing from regression trees [16] that have numeric values or decision trees that have classes at the leaf nodes. They can thus be compared to piecewise linear functions. Decision tree induction algorithm is used to build model tree. However, in contrast to decision tree where maximization of information gain is the splitting criteria, in model tree the minimization of variation (standard deviation – *sd*) in the intra-subset class values is used as the splitting criteria. At each node, the standard deviation of the class values reaching that node is taken as a measure of error. The expected reduction in error is then calculated for each attribute. The attribute which gives maximum reduction in error is chosen for performing splitting. The standard deviation reduction (SDR) is calculated by the following equation [15]:

$$\mathrm{SDR} = sd(K) - \sum_i \frac{|K_i|}{K} \times sd(K_i),$$

where $K$ is the set of examples that reach the node and $K_1, K_2, \ldots$ are the sets resulting from splitting the node according to the selected attribute.

The splitting stops if the change in class values (standard deviation) of instances reaching a node becomes very small or when there are only few instances left. Pruning is performed from the leaf nodes and inner nodes are transformed into leaf nodes by replacing them with a linear regression function. To avoid discontinuities between the linear models of the adjacent leaf nodes in the pruned tree, a smoothing process is carried out. The trees formed by M5 algorithm are generally much smaller than the regression trees. M5 model trees have the ability to efficiently scale with very high dimensional data having hundreds of attributes.

### 2.2. Sequential minimal optimization based support vector machine

Support vector machine (SVM) [17] has found applications in numerous fields for solving both regression and classification problems. It solves a nonlinear low dimensional classification problem by projecting it into high dimensional space where an optimal separating hyperplane is constructed between the positive and negative classes with maximum margin. Support vector regression tries to find a continuous value function that fits the data. The input is mapped to a high dimension feature space and a linear model is constructed in that space. The aim is to minimize the functions of the following form [18]:

$$L = \frac{l}{1}\sum_{i=1}^{l} |y_i - f(x_i, w, b)|_\varepsilon + \|w\|^2.$$

In the above equation $|\cdot|_\varepsilon$ is $\varepsilon$-sensitive error function defined as follows:

$$|x|_\varepsilon = \begin{cases} 0 & \text{if} \quad |x| < \varepsilon \\ |x| - \varepsilon & \text{otherwise} \end{cases}.$$

The output of SVM is then expressed as:

$$f(x, a^+, a^-, b) = \sum_{i=1}^{l} (a^+ - a^-) K(x_i, x) + b,$$

where $a_i^+$ and $a_i^-$ are positive and negative Lagrange multipliers that obey $0 \leqslant a_i^+, a_i^-, \forall_i$ and $a_i^+, a_i^-, = 0, \forall_i$.

Sequential minimal optimization is a learning algorithm for SVM originally proposed for classification problems [19]. A variant of this algorithm for support vector regression was proposed in [12] and subsequently improved by [13]. It continuously searches for two Lagrange multipliers that can be optimized with respect to each other and then computes the optimal step for the two Lagrange multipliers. For detailed explanation refer to [18].

## 3. Experimental dataset and results

The experimental dataset reported by [7] was chosen to study the performance of M5 model trees and support vector machine based regression models for predicting the minimum value of surface roughness. Their machining experiments involved 24 trials to measure $R_a$ value in the end milling process based on eight data of two levels DOE $2^k$ full factorial, four center and twelve axial points. All $R_a$ values were collected for three type of cutting tools: uncoated, TiA1N coated and SNTR coated. The experimental results reported by them are shown in Table 1 and are used in this paper for developing M5 and support vector machine based regression models.

It may be noted that for some instances in Table 1 (such as 9 and 10 or 13 and 14, etc.), identical cutting conditions result in different $R_a$ values. This discrepancy in $R_a$ values may be attributed to uncontrollable factors that affect machining such as tool wear, chips formation, vibrations, non-homogeneity of tool and work piece material and machine motion errors, etc. [20]. This implies that if further experiments are carried out, we may obtain further different $R_a$ values corresponding to identical cutting conditions, some of which may be lower than the currently known values. *The aim of this research is to build mathematical models that can predict such potential cutting conditions that may lead to least $R_a$ value.*

All the experimental data including cutting condition values as well as $R_a$ values of Table 1 were normalized using the following equation [3]:

$$x_i = \frac{0.8}{d_{\max} - d_{\min}} (d_i - d_{\min}) + 0.1. \tag{1}$$

Normalized data is shown in Table 2.

**Table 1**
Experimental cutting conditions and $R_a$ values for end milling process [7]. $R_a$ values are taken for three tools: uncoated, TiA1N coated and SNTR coated.

| No. | Data source | Setting values of experimental cutting conditions | | | Experimental $R_a$ value (μm) | | |
|---|---|---|---|---|---|---|---|
| | | *Cutting speed* $v$ (m/min) | *Feed rate* $f$ (mm/tooth) | Radial rake angle $\gamma$ (°) | $Ra_{\_\text{uncoated}}$ | $Ra_{\_\text{TiA1N}}$ | $Ra_{\_\text{SNTR}}$ |
| 1 | DOE $2^k$ | 130 | 0.03 | 7 | 0.365 | 0.32 | 0.284 |
| 2 | | 160 | 0.03 | 7 | 0.256 | 0.266 | 0.196 |
| 3 | | 130 | 0.07 | 7 | 0.498 | 0.606 | 0.668 |
| 4 | | 160 | 0.07 | 7 | 0.464 | 0.476 | 0.624 |
| 5 | | 130 | 0.03 | 13 | 0.428 | 0.260 | 0.280 |
| 6 | | 160 | 0.03 | 13 | 0.252 | 0.232 | 0.190 |
| 7 | | 130 | 0.07 | 13 | 0.561 | 0.412 | 0.612 |
| 8 | | 160 | 0.07 | 13 | 0.512 | 0.392 | 0.576 |
| 9 | Center | 144.22 | 0.046 | 9.5 | 0.464 | 0.324 | 0.329 |
| 10 | | 144.22 | 0.046 | 9.5 | 0.444 | 0.38 | 0.416 |
| 11 | | 144.22 | 0.046 | 9.5 | 0.448 | 0.460 | 0.352 |
| 12 | | 144.22 | 0.046 | 9.5 | 0.424 | 0.304 | 0.400 |
| 13 | Axial | 124.53 | 0.046 | 9.5 | 0.328 | 0.360 | 0.344 |
| 14 | | 124.53 | 0.046 | 9.5 | 0.324 | 0.308 | 0.320 |
| 15 | | 167.03 | 0.046 | 9.5 | 0.236 | 0.340 | 0.272 |
| 16 | | 167.03 | 0.046 | 9.5 | 0.240 | 0.356 | 0.288 |
| 17 | | 144.22 | 0.025 | 9.5 | 0.252 | 0.308 | 0.230 |
| 18 | | 144.22 | 0.025 | 9.5 | 0.262 | 0.328 | 0.234 |
| 19 | | 144.22 | 0.083 | 9.5 | 0.584 | 0.656 | 0.640 |
| 20 | | 144.22 | 0.083 | 9.5 | 0.656 | 0.584 | 0.696 |
| 21 | | 144.22 | 0.046 | 6.2 | 0.304 | 0.300 | 0.361 |
| 22 | | 144.22 | 0.046 | 6.2 | 0.288 | 0.316 | 0.360 |
| 23 | | 144.22 | 0.046 | 14.8 | 0.316 | 0.324 | 0.368 |
| 24 | | 144.22 | 0.046 | 14.8 | 0.348 | 0.396 | 0.360 |
| $R_a$ (minimum) | | | | | **0.236** | **0.232** | **0.190** |

### 3.1. Calculation process of predicted dataset

Calculation of predicted values from the available data using M5 and SMO-SVM involves the following steps:

1. The data is first divided into two sets: training set and test set. The decision of which data tuples should go in which set is very important and affects the accuracy of the predicated values. We have carefully selected the data tuples for each set differently than [7] and describe the rationale for this selection, later in this section.
2. The training data set is fed as input to the appropriate algorithm (M5 or SMO-SVM) which then generates the model of the data as output (cf. Sections 2.1 and 2.2).
3. The test set is supplied as input to the data model obtained in the previous step to get the predicted values.

This calculation process of predicted dataset is shown in Fig. 3. For experiments, in this paper, we divided the normalized data of Table 2 into two types of training and testing datasets. The composition of each of these datasets and the results obtained are discussed in the following sections.

### 3.2. Training and test dataset 1

In the first type of experiments, the training and testing datasets were created in the same way as [7] i.e. the last 16 rows of Table 2 were taken as training data as shown in Table 3 and the first 8 rows as the test data as shown in Table 4.

### 3.3. Results of M5P model tree based regression models for test dataset 1

Six model trees (3 cutting tools $\times$ 2 tree parameters: pruning enabled and pruning disabled) were built for the normalized training dataset of Table 3 using M5P tree implementation of the Weka suite of machine learning algorithms [21]. The results (predicted minimum $R_a$ value) are shown in Table 5.

To select the best M5 model tree, three factors: Root Mean Square Error (RMSE), correlation and minimum predicted value of $R_a$ are considered. Table 6 states the correlation and RMSE values of the six model trees corresponding to the testing data as reported by Weka. First we select the top three best model trees having the lowest RMSE values. Table 6 shows that the models for TiA1N cutting tool with pruning enabled, $_{\text{TiA1N}}$ cutting tool with pruning disabled and uncoated tool with pruning enabled have the lowest RMSE values of 0.2253, 0.2253 and 0.2257 respectively. Now, amongst these three selected models we consider the correlation value of each and select the best two having the highest correlation values. It is evident from Table 6 that the correlation values of the three models are the same i.e. 0. Thus, we select the models for TiA1N tool

**Table 2**
Normalized experimental cutting conditions and $R_a$ values for end milling process [7]. In contrast to [7] the first eight rows with all distinct cutting conditions pairs are taken as the training set. The remaining 16 rows are used as the test set.

| No. | Data source | Setting values of experimental cutting conditions | | | Experimental $R_a$ value (μm) | | |
|---|---|---|---|---|---|---|---|
| | | *Cutting speed* $v$ (m/min) | *Feed rate* $f$ (mm/tooth) | Radial rake angle $\gamma$ (°) | $Ra_{\_\text{uncoated}}$ | $Ra_{\_\text{TiA1N}}$ | $Ra_{\_\text{SNTR}}$ |
| 1 | DOE $2^k$ | 0.203 | 0.169 | 0.174 | 0.346 | 0.266 | 0.249 |
| 2 | | 0.768 | 0.169 | 0.174 | 0.138 | 0.164 | 0.109 |
| 3 | | 0.203 | 0.721 | 0.174 | 0.599 | 0.806 | 0.856 |
| 4 | | 0.768 | 0.721 | 0.174 | 0.534 | 0.560 | 0.786 |
| 5 | | 0.203 | 0.169 | 0.733 | 0.466 | 0.153 | 0.242 |
| 6 | | 0.768 | 0.169 | 0.733 | 0.130 | 0.100 | 0.100 |
| 7 | | 0.203 | 0.721 | 0.733 | 0.719 | 0.440 | 0.767 |
| 8 | | 0.768 | 0.721 | 0.733 | 0.626 | 0.402 | 0.710 |
| 9 | Center | 0.471 | 0.390 | 0.407 | 0.534 | 0.274 | 0.320 |
| 10 | | 0.471 | 0.390 | 0.407 | 0.496 | 0.379 | 0.457 |
| 11 | | 0.471 | 0.390 | 0.407 | 0.504 | 0.530 | 0.356 |
| 12 | | 0.471 | 0.390 | 0.407 | 0.458 | 0.236 | 0.432 |
| 13 | Axial | 0.100 | 0.390 | 0.407 | 0.275 | 0.342 | 0.343 |
| 14 | | 0.100 | 0.390 | 0.407 | 0.268 | 0.243 | 0.306 |
| 15 | | 0.900 | 0.390 | 0.407 | 0.100 | 0.304 | 0.230 |
| 16 | | 0.900 | 0.390 | 0.407 | 0.108 | 0.334 | 0.255 |
| 17 | | 0.471 | 0.100 | 0.407 | 0.130 | 0.243 | 0.163 |
| 18 | | 0.471 | 0.100 | 0.407 | 0.150 | 0.281 | 0.170 |
| 19 | | 0.471 | 0.900 | 0.407 | 0.763 | 0.900 | 0.811 |
| 20 | | 0.471 | 0.900 | 0.407 | 0.900 | 0.764 | 0.900 |
| 21 | | 0.471 | 0.390 | 0.100 | 0.230 | 0.228 | 0.370 |
| 22 | | 0.471 | 0.390 | 0.100 | 0.199 | 0.258 | 0.369 |
| 23 | | 0.471 | 0.390 | 0.900 | 0.252 | 0.274 | 0.381 |
| 24 | | 0.471 | 0.390 | 0.900 | 0.313 | 0.409 | 0.369 |
| $R_a$ (minimum) | | | | | **0.100** | **0.100** | **0.100** |

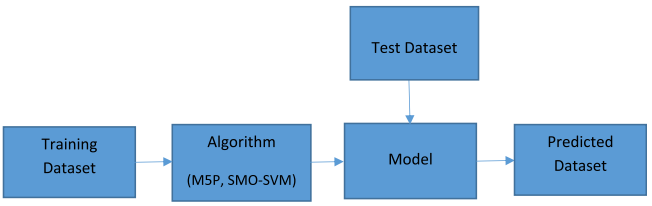


**Fig. 3.** Calculation process of the predicted dataset.

**Table 3**
Training dataset 1 used for generating models using M5 and SMO-SVM.

| No. | Data source | Setting values of experimental cutting conditions | | | Experimental $R_a$ value (μm) | | |
|---|---|---|---|---|---|---|---|
| | | $v$ (m/min) | $f$ (mm/tooth) | $\gamma$ (°) | $Ra_{\_uncoated}$ | $Ra_{\_TiA1N}$ | $Ra_{\_SNTR}$ |
| 1 | Center | 0.471 | 0.390 | 0.407 | 0.534 | 0.274 | 0.320 |
| 2 | | 0.471 | 0.390 | 0.407 | 0.496 | 0.379 | 0.457 |
| 3 | | 0.471 | 0.390 | 0.407 | 0.504 | 0.530 | 0.356 |
| 4 | | 0.471 | 0.390 | 0.407 | 0.458 | 0.236 | 0.432 |
| 5 | Axial | 0.100 | 0.390 | 0.407 | 0.275 | 0.342 | 0.343 |
| 6 | | 0.100 | 0.390 | 0.407 | 0.268 | 0.243 | 0.306 |
| 7 | | 0.900 | 0.390 | 0.407 | 0.100 | 0.304 | 0.230 |
| 8 | | 0.900 | 0.390 | 0.407 | 0.108 | 0.334 | 0.255 |
| 9 | | 0.471 | 0.100 | 0.407 | 0.130 | 0.243 | 0.163 |
| 10 | | 0.471 | 0.100 | 0.407 | 0.150 | 0.281 | 0.170 |
| 11 | | 0.471 | 0.900 | 0.407 | 0.763 | 0.900 | 0.811 |
| 12 | | 0.471 | 0.900 | 0.407 | 0.900 | 0.764 | 0.900 |
| 13 | | 0.471 | 0.390 | 0.100 | 0.230 | 0.228 | 0.370 |
| 14 | | 0.471 | 0.390 | 0.100 | 0.199 | 0.258 | 0.369 |
| 15 | | 0.471 | 0.390 | 0.900 | 0.252 | 0.274 | 0.381 |
| 16 | | 0.471 | 0.390 | 0.900 | 0.313 | 0.409 | 0.369 |

**Table 4**
Test dataset 1 used for generating prediction dataset.

| No. | Data source | Setting values of experimental cutting conditions | | | Experimental $R_a$ value (μm) | | |
|---|---|---|---|---|---|---|---|
| | | $v$ (m/min) | $f$ (mm/tooth) | $\gamma$ (°) | $Ra_{\_uncoated}$ | $Ra_{\_TiA1N}$ | $Ra_{\_SNTR}$ |
| 1 | DOE $2^k$ | 0.203 | 0.169 | 0.174 | 0.346 | 0.266 | 0.249 |
| 2 | | 0.768 | 0.169 | 0.174 | 0.138 | 0.164 | 0.109 |
| 3 | | 0.203 | 0.721 | 0.174 | 0.599 | 0.806 | 0.856 |
| 4 | | 0.768 | 0.721 | 0.174 | 0.534 | 0.560 | 0.786 |
| 5 | | 0.203 | 0.169 | 0.733 | 0.466 | 0.153 | 0.242 |
| 6 | | 0.768 | 0.169 | 0.733 | 0.130 | 0.100 | 0.100 |
| 7 | | 0.203 | 0.721 | 0.733 | 0.719 | 0.440 | 0.767 |
| 8 | | 0.768 | 0.721 | 0.733 | 0.626 | 0.402 | 0.710 |

**Table 5**
Predicted $R_a$ values of M5 model tress for the three tools in test dataset 1. Two models are developed for each tool, one with pruning enabled and the other with pruning disabled.

| No. | Data source | Unpruned = true | | | Unpruned = false | | |
|---|---|---|---|---|---|---|---|
| | | $R_a$_uncoated | $R_a$_TiA1N | $R_a$ SNTR | $R_a$_uncoated | $R_a$_TiA1N | $R_a$ SNTR |
| 1 | DOE $2^k$ | 0.355 | 0.375 | 0.39 | 0.355 | 0.375 | 0.39 |
| 2 | | 0.355 | 0.375 | 0.39 | 0.355 | 0.375 | 0.39 |
| 3 | | 0.355 | 0.375 | 0.39 | 0.355 | 0.375 | 0.39 |
| 4 | | 0.355 | 0.375 | 0.39 | 0.355 | 0.375 | 0.39 |
| 5 | | 0.355 | 0.375 | 0.39 | 0.355 | 0.375 | 0.39 |
| 6 | | 0.355 | 0.375 | 0.39 | 0.355 | 0.375 | 0.39 |
| 7 | | 0.355 | 0.375 | 0.39 | 0.355 | 0.375 | 0.39 |
| 8 | | 0.355 | 0.375 | 0.39 | 0.355 | 0.375 | 0.39 |
| *Ra* (minimum) | | **0.355** | **0.375** | **0.39** | **0.355** | **0.375** | **0.39** |

**Table 6**
Correlation and Root Mean Square Error values of model trees for the test dataset 1.

| No. | Modeling technique | $R_{a\_uncoated}$ | | $R_{a\_TiA1N}$ | | $R_{a\_SNTR}$ | |
|---|---|---|---|---|---|---|---|
| | | RMSE | Correlation | RMSE | Correlation | RMSE | Correlation |
| 1 | $M5_{Unpruned=True}$ | 0.2257 | 0 | 0.2253 | 0 | 0.3209 | 0 |
| 2 | $M5_{Unpruned=False}$ | 0.2257 | 0 | 0.2253 | 0 | 0.3209 | 0 |

**Table 7**
Predicted $R_a$ values of SMO-SVM models for the three tools in test dataset 1. Two models are developed for each tool, one with normalized polynomial kernel and the other with RBF kernel. The values of exponent and gamma shown are the ones which resulted in least RMSE.

| No. | Data source | Normalized polynomial kernel | | | RBF kernel | | |
|---|---|---|---|---|---|---|---|
| | | $R_a$_uncoated | $R_a$_TiA1N | $R_a$ SNTR | $R_a$_uncoated | $R_a$_TiA1N | $R_a$ SNTR |
| | | Exponent = 1.10 | Exponent = 8 | 5.1 | Gamma = 3.91 | Gamma = 7 | Gamma = 0.7 |
| 1 | DOE $2^k$ | 0.315 | 0.38 | 0.435 | 0.089 | 0.305 | 0.22 |
| 2 | | 0.003 | 0.362 | 0.21 | 0.023 | 0.309 | 0.166 |
| 3 | | 0.444 | 0.802 | 0.906 | 0.456 | 0.482 | 0.637 |
| 4 | | 0.26 | 0.309 | 0.44 | 0.383 | 0.47 | 0.58 |
| 5 | | 0.453 | 0.362 | 0.372 | 0.483 | 0.488 | 0.625 |
| 6 | | 0.149 | 0.271 | 0.187 | 0.144 | 0.376 | 0.17 |
| 7 | | 0.453 | 0.362 | 0.372 | 0.483 | 0.488 | 0.625 |
| 8 | | 0.339 | 0.406 | 0.471 | 0.418 | 0.477 | 0.57 |
| *Ra* (minimum) | | **0.003** | **0.271** | **0.187** | **0.023** | **0.305** | **0.166** |

with pruning disabled and TiA1N tool with pruning enabled as the best two models and proceed by considering the third factor, minimum predicted value of $R_a$. Table 5 shows that the predicted minimum values of $R_a$ by TiA1N tool with pruning disabled is 0.375 whereas for TiA1N tool with pruning enabled it is also 0.375. *Thus, we select* TiA1N *tool with pruning enabled as the best prediction model* (TiA1N tool with pruning disabled can also be equally selected as the best model as it is also having the similar RMSE, correlation and minimum predicted value of $R_a$).

The minimum normalized predicted $R_a$ value by the selected best M5 model tree is 0.375. The corresponding normalized and actual cutting conditions values cannot be determined explicitly as the generated model has predicted this constant value 0.375 with respect to all input values. This is not a useful and interesting result as the predicted value is higher than the experimental value which is in contrast to the goals of this work i.e. to determine a lower $R_a$ value and the corresponding input cutting conditions. The denormalized minimum predicted $R_a$ value is calculated using a modified Eq. (1) as follows [7]:

$$d_i = \frac{(y_{Ra} - 0.100)(d_{\max} - d_{\min})}{0.8} + d_{\min} = \frac{(0.375 - 0.100)(0.696 - 0.190)}{0.8} + 0.190 \approx 0.364\ \mu\text{m}.$$

### 3.4. Results of SMO-SVM models for test dataset 1

Six SVM models (3 cutting tools × 2 SVM kernels) were developed using SMO-SVM implementation of Weka (SMOreg) with the normalized training dataset of Table 3. Table 7 shows the result (prediction of minimum $R_a$ value) of applying the SMO-SVM models to the test set. Two kernels: normalized polynomial kernel and radial basis function (RBF) kernel were used for each of the cutting tool data. Experiments were performed with different values of Exponent parameter for normalized polynomial kernel and Gamma parameter for RBF kernel and the values resulting in lowest RMSE were selected finally and reported in Table 7.

To determine the best SMO-SVM prediction model, three factors: Root Mean Square Error (RMSE), correlation and minimum predicted value of $R_a$ are considered. Table 8 states the correlation and RMSE values of the six SMO-SVM models corresponding to the testing data as reported by Weka. First we select the top three best SMO-SVM models having the lowest RMSE values. Table 8 shows that the models for TiA1N cutting tool with normalized polynomial kernel and uncoated cutting tool with RBF kernel and normalized polynomial kernel have the lowest RMSE values of 0.1558, 0.1667 and 0.1843 respectively. Now, amongst these three selected models we consider the correlation value of each and select the best two having the highest correlation values. It is evident from Table 8 that the correlation values of the three models are 0.7519, 0.882 and 0.8329 respectively. Thus, we select the models for uncoated tool with RBF kernel and normalized polynomial kernel as the best two models and proceed by considering the third factor, minimum predicted value of $R_a$. Table 7 shows that the predicted minimum values of $R_a$ by uncoated tool with RBF kernel is 0.023 whereas for normalized polynomial kernel it is 0.003. *Thus, we select uncoated tool with normalized polynomial kernel as the best prediction model.*

The minimum normalized predicted $R_a$ value by the selected best SMO-SVM model is 0.003 (row two of Table 7) and the corresponding normalized cutting conditions values are: $v = 0.768$, $f = 0.169$ and $\gamma = 0.174$ (second row of Table 4). The actual

**Table 8**
Correlation and Root Mean Square Error values of SMO-SVM models for the test dataset 1.

| No. | Modeling Technique | $R_{a\_uncoated}$ | | $R_{a\_TiA1N}$ | | $R_{a\_SNTR}$ | |
|---|---|---|---|---|---|---|---|
| | | RMSE | Correlation | RMSE | Correlation | RMSE | Correlation |
| 1 | SMO-SVM (normalized poly kernel) | 0.1843 | 0.8329 | 0.1558 | 0.7519 | 0.2249 | 0.7073 |
| 2 | SMO-SVM (RBF kernel) | 0.1667 | 0.882 | 0.2037 | 0.5512 | 0.1889 | 0.8086 |

**Table 9**
Training dataset 2 used for generating models using M5 and SMO-SVM. In contrast to the first type of experiments and [7] where the training set contained duplicate tuples, here all the eight rows are distinct tuples having dissimilar cutting conditions pairs.

| No. | Data source | Setting values of experimental cutting conditions | | | Experimental $R_a$ value (μm) | | |
|---|---|---|---|---|---|---|---|
| | | $v$ (m/min) | $f$ (mm/tooth) | $\gamma$ (°) | $Ra_{\_uncoated}$ | $Ra_{\_TiA1N}$ | $Ra_{\_SNTR}$ |
| 1 | DOE $2^k$ | 0.203 | 0.169 | 0.174 | 0.346 | 0.266 | 0.249 |
| 2 | | 0.768 | 0.169 | 0.174 | 0.138 | 0.164 | 0.109 |
| 3 | | 0.203 | 0.721 | 0.174 | 0.599 | 0.806 | 0.856 |
| 4 | | 0.768 | 0.721 | 0.174 | 0.534 | 0.560 | 0.786 |
| 5 | | 0.203 | 0.169 | 0.733 | 0.466 | 0.153 | 0.242 |
| 6 | | 0.768 | 0.169 | 0.733 | 0.130 | 0.100 | 0.100 |
| 7 | | 0.203 | 0.721 | 0.733 | 0.719 | 0.440 | 0.767 |
| 8 | | 0.768 | 0.721 | 0.733 | 0.626 | 0.402 | 0.710 |

**Table 10**
Test dataset 2 used for generating prediction dataset.

| No. | Data source | Setting values of experimental cutting conditions | | | Experimental $R_a$ value (μm) | | |
|---|---|---|---|---|---|---|---|
| | | $v$ (m/min) | $f$ (mm/tooth) | $\gamma$ (°) | $Ra_{\_uncoated}$ | $Ra_{\_TiA1N}$ | $Ra_{\_SNTR}$ |
| 1 | Center | 0.471 | 0.390 | 0.407 | 0.534 | 0.274 | 0.320 |
| 2 | | 0.471 | 0.390 | 0.407 | 0.496 | 0.379 | 0.457 |
| 3 | | 0.471 | 0.390 | 0.407 | 0.504 | 0.530 | 0.356 |
| 4 | | 0.471 | 0.390 | 0.407 | 0.458 | 0.236 | 0.432 |
| 5 | Axial | 0.100 | 0.390 | 0.407 | 0.275 | 0.342 | 0.343 |
| 6 | | 0.100 | 0.390 | 0.407 | 0.268 | 0.243 | 0.306 |
| 7 | | 0.900 | 0.390 | 0.407 | 0.100 | 0.304 | 0.230 |
| 8 | | 0.900 | 0.390 | 0.407 | 0.108 | 0.334 | 0.255 |
| 9 | | 0.471 | 0.100 | 0.407 | 0.130 | 0.243 | 0.163 |
| 10 | | 0.471 | 0.100 | 0.407 | 0.150 | 0.281 | 0.170 |
| 11 | | 0.471 | 0.900 | 0.407 | 0.763 | 0.900 | 0.811 |
| 12 | | 0.471 | 0.900 | 0.407 | 0.900 | 0.764 | 0.900 |
| 13 | | 0.471 | 0.390 | 0.100 | 0.230 | 0.228 | 0.370 |
| 14 | | 0.471 | 0.390 | 0.100 | 0.199 | 0.258 | 0.369 |
| 15 | | 0.471 | 0.390 | 0.900 | 0.252 | 0.274 | 0.381 |
| 16 | | 0.471 | 0.390 | 0.900 | 0.313 | 0.409 | 0.369 |

cutting condition values are $v = 160$ m/min, $f = 0.03$ mm/tooth and $\gamma = 7°$ (second row of Table 1). The denormalized minimum predicted $R_a$ value is calculated using a modified Eq. (1) as follows [7]:

$$d_i = \frac{(y_{Ra} - 0.100)(d_{\max} - d_{\min})}{0.8} + d_{\min} = \frac{(0.003 - 0.100)(0.696 - 0.190)}{0.8} + 0.190 \approx 0.128\ \mu\text{m}.$$

### 3.5. Training and test dataset 2

In the second type of experiments, the training and testing datasets were created differently by selecting different tuples than the first type. The normalized data of Table 2 was divided into training and test sets as follows: DOE $2^k$ data (first eight rows of Table 2) were used as the training set as shown in Table 9, the remaining 16 tuples of data (the four tuples of center data and twelve tuples of axial data) were used as the test set as shown in Table 10. This is in contrast to the first type of experiments and [7] where the last 16 rows of Table 2 were taken as training data and the first 8 rows as the test data. Our rationale for this division is as follows: the first eight rows have distinct pairs of cutting conditions values; however, the last 16 rows have instances where identical cutting conditions have resulted in different $R_a$ values. The data with distinct input and output values will give more opportunity to the training algorithm to better understand the characteristics of the

**Table 11**
Predicted $R_a$ values of M5 model tress for the three tools in test dataset 2. Two models are developed for each tool, one with pruning enabled and the other with pruning disabled.

| No. | Data source | Pruning disabled | | | Pruning enabled | | |
|---|---|---|---|---|---|---|---|
| | | $R_a$_uncoated | $R_a$_TiA1N | $R_a$ SNTR | $R_a$_uncoated | $R_a$_TiA1N | $R_a$ SNTR |
| 1 | Center | 0.333 | 0.280 | 0.326 | 0.358 | 0.301 | 0.361 |
| 2 | | 0.333 | 0.280 | 0.326 | 0.358 | 0.301 | 0.361 |
| 3 | | 0.333 | 0.280 | 0.326 | 0.358 | 0.301 | 0.361 |
| 4 | | 0.333 | 0.280 | 0.326 | 0.358 | 0.301 | 0.361 |
| 5 | Axial | 0.333 | 0.280 | 0.326 | 0.358 | 0.301 | 0.361 |
| 6 | | 0.333 | 0.280 | 0.326 | 0.358 | 0.301 | 0.361 |
| 7 | | 0.333 | 0.280 | 0.326 | 0.358 | 0.301 | 0.361 |
| 8 | | 0.333 | 0.280 | 0.326 | 0.358 | 0.301 | 0.361 |
| 9 | | 0.165 | 0.120 | 0.099 | 0.157 | 0.109 | 0.088 |
| 10 | | 0.165 | 0.120 | 0.099 | 0.157 | 0.109 | 0.088 |
| 11 | | 0.682 | 0.618 | 0.803 | 0.713 | 0.640 | 0.841 |
| 12 | | 0.682 | 0.618 | 0.803 | 0.713 | 0.640 | 0.841 |
| 13 | | 0.333 | 0.403 | 0.326 | 0.358 | 0.448 | 0.361 |
| 14 | | 0.333 | 0.403 | 0.326 | 0.358 | 0.448 | 0.361 |
| 15 | | 0.333 | 0.084 | 0.326 | 0.358 | 0.066 | 0.361 |
| 16 | | 0.333 | 0.084 | 0.326 | 0.358 | 0.066 | 0.361 |
| $R_a$ (minimum) | | **0.165** | **0.084** | **0.099** | **0.157** | **0.066** | **0.088** |

**Table 12**
Correlation and Root Mean Square Error values of model trees for the test dataset 2.

| No. | Modeling technique | $R_{a\_uncoated}$ | | $R_{a\_TiA1N}$ | | $R_{a\_SNTR}$ | |
|---|---|---|---|---|---|---|---|
| | | RMSE | Correlation | RMSE | Correlation | RMSE | Correlation |
| 1 | $M5_{Prunning\ Disabled}$ | 0.1404 | 0.8209 | 0.1614 | 0.6917 | 0.0669 | 0.9534 |
| 2 | $M5_{Prunning\ Enabled}$ | 0.1412 | 0.8136 | 0.1669 | 0.6424 | 0.0638 | 0.946 |

data and build more accurate classifier than the data having identical input values but different output values. Thus, the first 8 rows of Table 2 (with all distinct cutting conditions pairs) are selected as training data and the remaining 16 rows (with repeated identical cutting conditions pairs) as the testing data.

### 3.6. Results of M5P model tree based regression models for test dataset 2

Six model trees (3 cutting tools × 2 tree parameters: pruning enabled and pruning disabled) were built for the normalized training dataset of Table 3 using M5P tree implementation of the Weka suite of machine learning algorithms [21]. The results (predicted minimum $R_a$ value) are shown in Table 11.

To select the best M5 model tree, three factors: Root Mean Square Error (RMSE), correlation and minimum predicted value of $R_a$ are considered. Table 12 states the correlation and RMSE values of the six model trees corresponding to the testing data as reported by Weka. First we select the top three best model trees having the lowest RMSE values. Table 12 shows that the models for SNTR cutting tool with pruning enabled, SNTR cutting tool with pruning disabled and uncoated tool with pruning disabled have the lowest RMSE values of 0.0638, 0.0669 and 0.1404 respectively. Now, amongst these three selected models we consider the correlation value of each and select the best two having the highest correlation values. It is evident from Table 12 that the correlation values of the three models are 0.946, 0.9534 and 0.8209 respectively. Thus, we select the models for SNTR tool with pruning disabled and SNTR tool with pruning enabled as the best two models and proceed by considering the third factor, minimum predicted value of $R_a$. Table 11 shows that the predicted minimum values of $R_a$ by SNTR tool with pruning disabled is 0.099 whereas for SNTR tool with pruning enabled it is 0.088. *Thus, we select SNTR tool with pruning enabled as the best prediction model.*

The minimum normalized predicted $R_a$ value by the selected best M5 model tree is 0.088 (row nine and ten of Table 11) and the corresponding normalized cutting conditions values are: $v = 0.471$, $f = 0.100$ and $\gamma = 0.407$ (ninth and tenth rows of Table 10). The actual cutting condition values are $v = 144.22$ m/min, $f = 0.025$ mm/tooth and $\gamma = 9.5°$ (seventeenth and eighteenth rows of Table 1). The denormalized minimum predicted $R_a$ value is calculated using a modified Eq. (1) is as follows [7]:

$$d_i = \frac{(y_{Ra} - 0.100)(d_{\max} - d_{\min})}{0.8} + d_{\min} = \frac{(0.088 - 0.100)(0.696 - 0.190)}{0.8} + 0.190 \approx 0.182\ \mu\text{m}.$$

**Table 13**
Predicted $R_a$ values of SMO-SVM models for the three tools in test dataset 2. Two models are developed for each tool, one with normalized polynomial kernel and the other with RBF kernel. The values of exponent and gamma shown are the ones which resulted in least RMSE.

| No. | Data source | Normalized polynomial kernel | | | RBF kernel | | |
|---|---|---|---|---|---|---|---|
| | | $R_a$_uncoated | $R_a$_TiA1 N | $R_a$ SNTR | $R_a$_uncoated | $R_a$_TiA1 N | $R_a$ SNTR |
| | | Exponent = 9 | Exponent = 8 | Exponent = 1.55 | Gamma = 1 | Gamma = 0.5 | Gamma = 4.9 |
| 1 | Center | 0.360 | 0.324 | 0.423 | 0.372 | 0.318 | 0.393 |
| 2 | | 0.360 | 0.324 | 0.423 | 0.372 | 0.318 | 0.393 |
| 3 | | 0.360 | 0.324 | 0.423 | 0.372 | 0.318 | 0.393 |
| 4 | | 0.360 | 0.324 | 0.423 | 0.372 | 0.318 | 0.393 |
| 5 | Axial | 0.494 | 0.426 | 0.630 | 0.469 | 0.361 | 0.481 |
| 6 | | 0.494 | 0.426 | 0.630 | 0.469 | 0.361 | 0.481 |
| 7 | | 0.158 | 0.160 | 0.244 | 0.273 | 0.272 | 0.335 |
| 8 | | 0.158 | 0.160 | 0.244 | 0.273 | 0.272 | 0.335 |
| 9 | | 0.113 | 0.088 | 0.094 | 0.171 | 0.136 | 0.117 |
| 10 | | 0.113 | 0.088 | 0.094 | 0.171 | 0.136 | 0.117 |
| 11 | | 0.635 | 0.740 | 0.735 | 0.712 | 0.630 | 0.821 |
| 12 | | 0.635 | 0.740 | 0.735 | 0.712 | 0.630 | 0.821 |
| 13 | | 0.486 | 0.498 | 0.520 | 0.370 | 0.392 | 0.426 |
| 14 | | 0.486 | 0.498 | 0.520 | 0.370 | 0.392 | 0.426 |
| 15 | | 0.344 | 0.271 | 0.328 | 0.388 | 0.208 | 0.392 |
| 16 | | 0.344 | 0.271 | 0.328 | 0.388 | 0.208 | 0.392 |
| *Ra* (minimum) | | **0.113** | **0.088** | **0.094** | **0.171** | **0.136** | **0.117** |

**Table 14**
Correlation and Root Mean Square Error values of SMO-SVM models for the test dataset 2.

| No. | Modeling technique | $R_{a\_uncoated}$ | | $R_{a\_TiA1N}$ | | $R_{a\_SNTR}$ | |
|---|---|---|---|---|---|---|---|
| | | RMSE | Correlation | RMSE | Correlation | RMSE | Correlation |
| 1 | SMO-SVM (normalized poly kernel) | 0.1639 | 0.6984 | 0.1551 | 0.6752 | 0.1364 | 0.769 |
| 2 | SMO-SVM (RBF kernel) | 0.1403 | 0.8243 | 0.1346 | 0.7463 | 0.0780 | 0.9290 |

### 3.7. Results of SMO-SVM based regression models for test dataset 2

Six SVM models (3 cutting tools × 2 SVM kernels) were developed using SMO-SVM implementation of Weka (SMOreg) with the normalized training dataset of Table 9. Table 13 shows the result (prediction of minimum $R_a$ value) of applying the SMO-SVM models to the test dataset 2. Two kernels: normalized polynomial kernel and radial basis function (RBF) kernel were used for each of the cutting tool data. Experiments were performed with different values of Exponent parameter for normalized polynomial kernel and Gamma parameter for RBF Kernel and the values resulting in lowest RMSE were selected finally and reported in Table 13.

To determine the best SMO-SVM prediction model, three factors: Root Mean Square Error (RMSE), correlation and minimum predicted value of $R_a$ are considered. Table 14 states the correlation and RMSE values of the six SMO-SVM models corresponding to the testing data as reported by Weka. First we select the top three best SMO-SVM models having the lowest RMSE values. Table 14 shows that the models for SNTR cutting tool and TiA1N cutting tool with RBF kernel and SNTR tool with normalized polynomial kernel have the lowest RMSE values of 0.0780, 0.1346 and 0.1364 respectively. Now, amongst these three selected models we consider the correlation value of each and select the best two having the highest correlation values. It is evident from Table 14 that the correlation values of the three models are 0.9290, 0.7463 and 0.769 respectively. Thus, we select the models for SNTR tool with RBF kernel and normalized polynomial kernel as the best two models and proceed by considering the third factor, minimum predicted value of $R_a$. Table 13 shows that the predicted minimum values of $R_a$ by SNTR tool with RBF kernel is 0.117 whereas for normalized polynomial kernel it is 0.094. *Thus, we select SNTR tool with Normalized Polynomial kernel as the best prediction model.*

The minimum normalized predicted $R_a$ value by the selected best SMO-SVM model is 0.094 (row nine and ten of Table 13) and the corresponding normalized cutting conditions values are: $v = 0.471$, $f = 0.100$ and $\gamma = 0.407$ (ninth and tenth rows of Table 10). The actual cutting condition values are $v = 144.22$ m/min, $f = 0.025$ mm/tooth and $\gamma = 9.5°$ (seventeenth and eighteenth rows of Table 1). The denormalized minimum predicted $R_a$ value is calculated using a modified Eq. (1) as follows [7]:

$$d_i = \frac{(y_{Ra} - 0.100)(d_{\max} - d_{\min})}{0.8} + d_{\min} = \frac{(0.094 - 0.100)(0.696 - 0.190)}{0.8} + 0.190 \approx 0.186\ \mu\text{m}.$$

## 4. Evaluation of results

The validation of model tree and SMO-SVM based models for predicting minimum value of $R_a$ is carried out by plotting and observing the graphs of predicted values vs. experimental values and using paired-samples $t$ tests.

Figs. 4–6 show the plots of predicted values by M5 model trees vs. experimental values for test dataset 1. It is evident from the Figs. 4–6 that in case of model trees applied on test dataset 1, the predicted values remain constant for all data

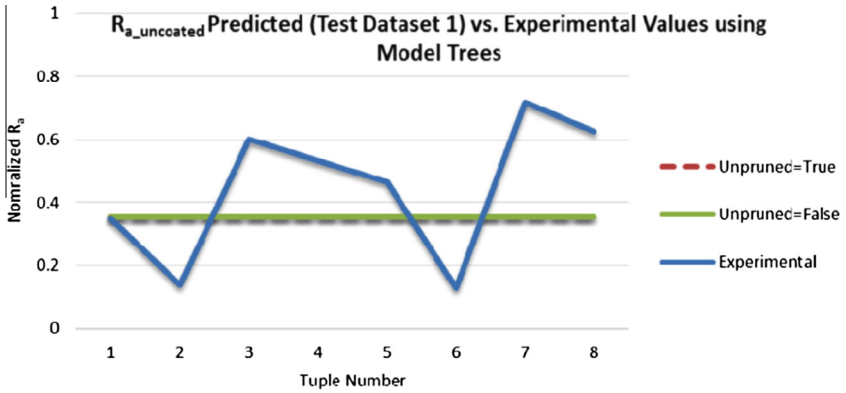


**Fig. 4.** Test dataset 1 (uncoated tool), predicted $R_a$ by model trees vs. experimental data.

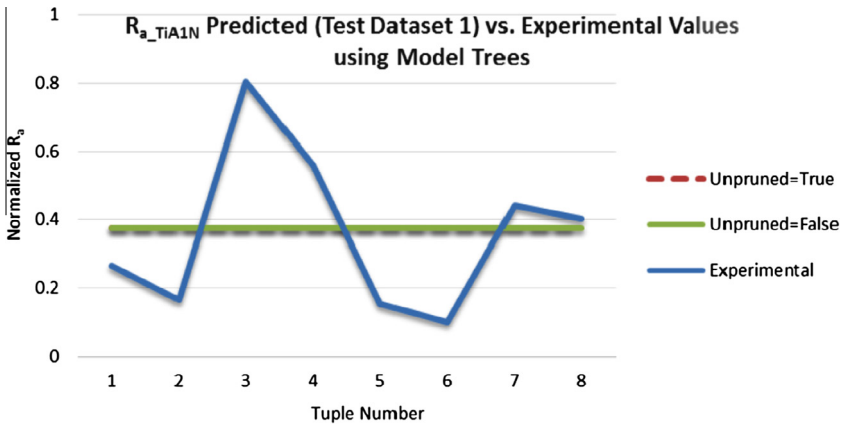


**Fig. 5.** Test dataset 1 (TiA1N tool), predicted $R_a$ by model trees vs. experimental data.

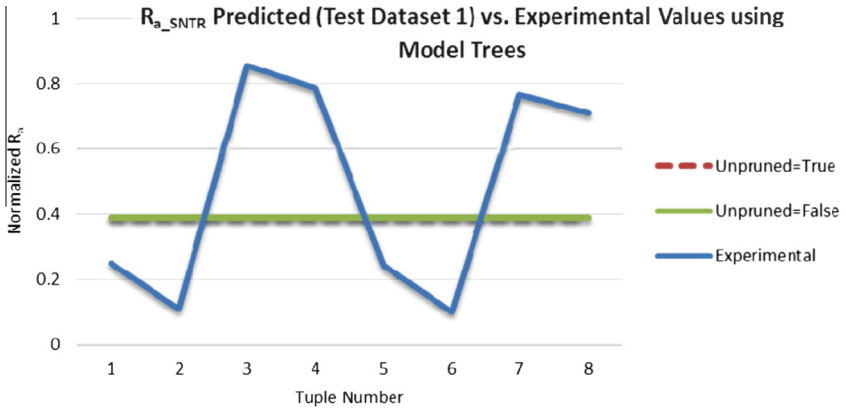


**Fig. 6.** Test dataset 1 (SNTR tool), predicted $R_a$ by model trees vs. experimental data.

**Table 15**
Statistics and correlations (experimental data vs. best M5 model trees for test dataset 1). Correlation value zero implies that the model does not bear any relationship with the experimental data.

| | Mean | *N* | Std. Deviation | Std. Error mean | Correlation |
|---|---|---|---|---|---|
| Experimental_TiA1N | 0.3614 | 8 | 0.24047 | 0.08502 | 0 |
| $M5_{Pruning_enabled_}$TiA1N | 0.3750 | 8 | 0.00000 | 0.00000 | |

**Table 16**
Paired-samples *t* test (experimental data vs. best M5 model tree for test dataset 1).

| Pair | Paired differences | | | | | *t* | *df* | Sig. (2-tailed) |
|---|---|---|---|---|---|---|---|---|
| | | | | 95% Confidence interval of the difference | | | | |
| | Mean | Std. Deviation | Std. Error mean | Lower | Upper | | | |
| Experimental_TiA1N and $M5_{Unpruned=True\_TiA1N}$ | −0.01363 | 0.240466 | 0.085017 | −0.21466 | 0.1874 | −0.1603 | 7 | 0.877 |

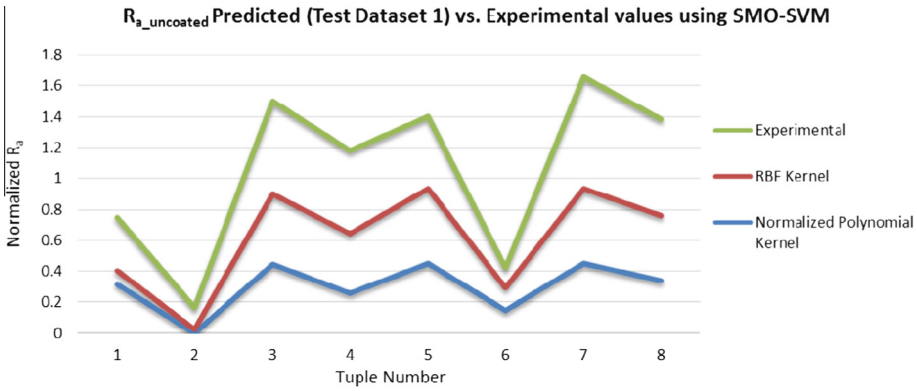


**Fig. 7.** Test dataset 1 (uncoated tool), predicted $R_a$ by SMO-SVM vs. experimental data.

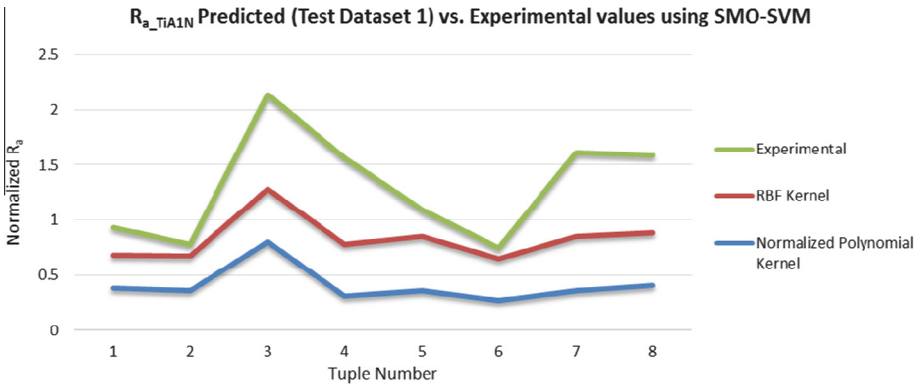


**Fig. 8.** Test dataset 1 (TiA1N tool), predicted $R_a$ by SMO-SVM vs. experimental data.

tuples resulting in the straight lines as compared to the experimental values which fluctuate widely and are represented by the zigzag lines. This high dissimilarity in the line patterns indicate that the predicted values by model tree for test dataset 1 does not follow the experimental values adequately. Tables 15 and 16 present the results of paired-samples *t* test of test dataset 1 experimental data for TiA1N tool paired with the predicated data by the best model tree (TiA1N tool with pruning enabled). Tables 15 and 16 prove that the mean $R_a$ value is increased by 0.01363 from the experimental results for TiA1N tool to the best model tree results, $t(7) = -0.1603$ and $p = 0.877$. The 95% confidence interval ranges from −0.03224 to 0.1874. Thus, the two means of experimental TiA1N tool and the best model tree are not significantly different from each other.

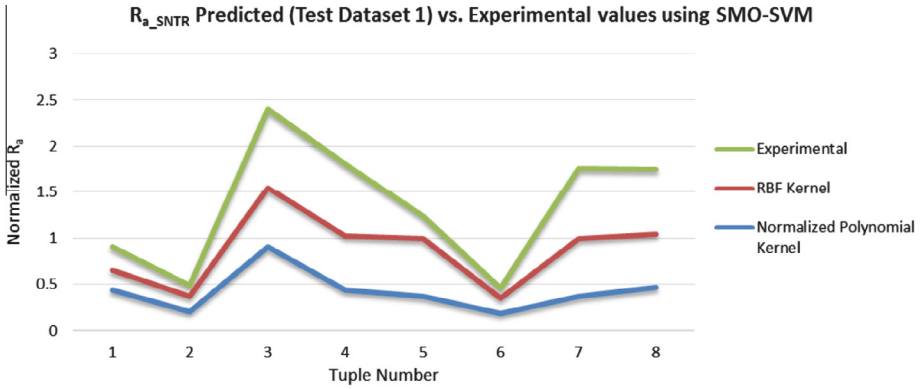


**Fig. 9.** Test dataset 1 (SNTR tool), predicted $R_a$ by SMO-SVM vs. experimental data.

**Table 17**
Statistics and correlations (experimental data vs. best SMO-SVM model for test dataset 1).

| | Mean | *N* | Std. Deviation | Std. Error mean | Correlation |
|---|---|---|---|---|---|
| Experimental_Uncoated | 0.448 | 8 | 0.22142 | 0.07828 | 0.833 |
| SMO-SVM_NormPoly_Uncoated | 0.3020 | 8 | 0.30200 | 0.161191 | |

**Table 18**
Paired-samples *t* test (experimental data vs. best SMO-SVM for test dataset 1).

| Pair | Paired differences | | | | | *t* | *df* | Sig. (2-tailed) |
|---|---|---|---|---|---|---|---|---|
| | | | | 95% Confidence interval of the difference | | | | |
| | Mean | Std. Deviation | Std. Error mean | Lower | Upper | | | |
| Experimental_Uncoated and SMO-SVM_NormPoly_Uncoated | 0.14275 | 0.124741 | 0.044102 | 0.038464 | 0.247 | 3.23678 | 7 | 0.014 |

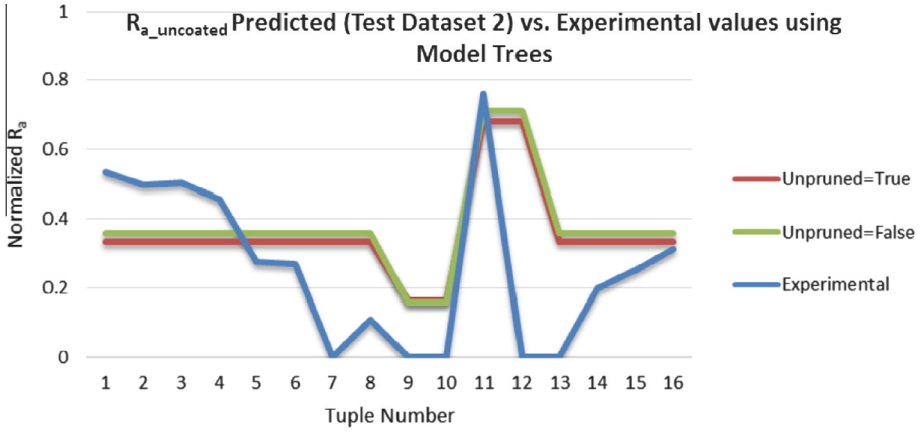


**Fig. 10.** Test dataset 2 (uncoated tool), predicted $R_a$ by model trees vs. experimental data.

However, the correlation value of zero indicates that the generated model bears no relationship with the real experimental data.

Figs. 7–9 show the plots of predicted values by SMO-SVM vs. experimental values for test dataset 1. It is evident from the Figs. 7–9 that in case of SMO-SVM applied on test dataset 1, the predicted values fluctuate in the similar manner as the

experimental values thus resulting in similar line patterns. Tables 17 and 18 present the results of paired-samples $t$ test of test dataset 1 experimental data for uncoated tool paired with the predicated data by the best SMO-SVM model (uncoated tool with normalized polynomial kernel). Tables 17 and 18 prove that the mean $R_a$ value is decreased by 0.14275 from the experimental results for uncoated tool to the best SMO-SVM model, $t\,(7) = 3.23678$ and $p = 0.014$. The 95% confidence interval ranges from 0.038464 to 0.247. Thus, the two means of experimental uncoated tool and the best SMO-SVM model are significantly different from each other indicating that the generated model does not bears good relationship with the real experimental data.

Figs. 10–12 show the plots of predicted values by model tree vs. experimental values for test dataset 2. It is evident from the Figs. 10–12 that in case of model tree applied on test dataset 2, the predicted values fluctuate in the similar manner as the experimental values thus resulting in similar line patterns. Tables 19 and 20 present the results of paired-samples $t$ test of test dataset 2 experimental data for SNTR tool paired with the predicated data by the best model tree (SNTR tool with pruning enabled). Tables 19 and 20 prove that the mean $Ra$ value is reduced by 0.0026 from experimental results for SNTR tool to the best model tree results, $t\,(15) = 0.16$ and $p = 0.875$. The 95% confidence interval ranges from −0.21466 to 0.03769. Thus, the two means of experimental SNTR tool and the best model tree are not significantly different from each other.

Figs. 12–15 show the plots of predicted values by SMO-SVM vs. experimental values for test dataset 2. It is evident from the Figs. 12–15 that in case of SMO-SVM applied on test dataset 2, the predicted values fluctuate in the similar manner as the experimental values thus resulting in similar line patterns. Tables 21 and 22 show the results of paired-samples $t$ test of test dataset 2 experimental data for SNTR tool paired with the predicated data by the best SMO-SVM model (SNTR tool with normalized polynomial kernel). Tables 21 and 22 show that the mean $R_a$ value is increased by 0.035 from experimental results for SNTR tool to the best SMO-SVM results, the 95% confidence interval ranges from −0.03224 to 0.03769, $t\,(15) = -1.033$

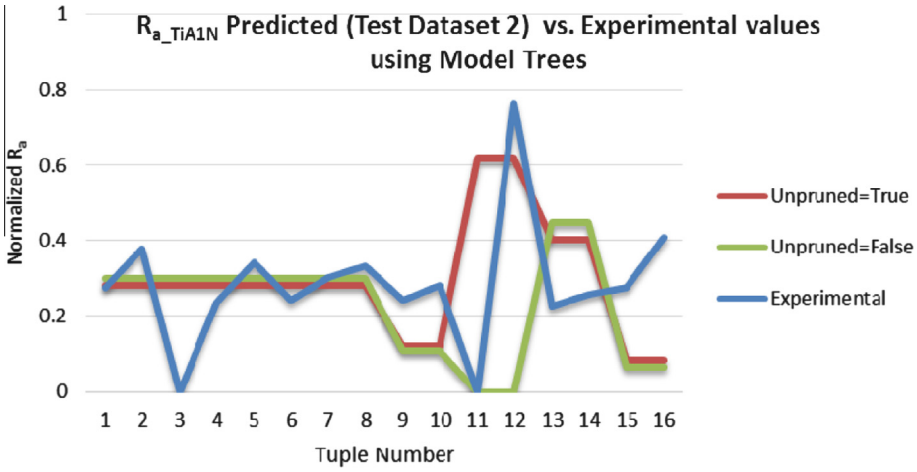


**Fig. 11.** Test dataset 2 (TiA1N tool), predicted $R_a$ by model trees vs. experimental data.

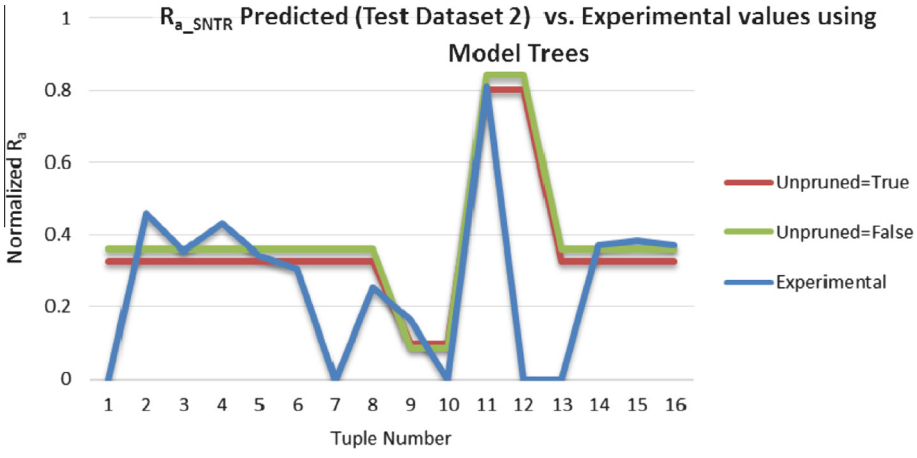


**Fig. 12.** Test dataset 2 (SNTR tool), predicted $R_a$ by model trees vs. experimental data.

**Table 19**
Statistics and correlations (experimental data vs. best M5 model tree for test dataset 2).

| | Mean | N | Std. Deviation | Std. Error mean | Correlation |
|---|---|---|---|---|---|
| Experimental_SNTR | 0.3895 | 16 | 0.20067 | 0.050168 | 0.946 |
| $M5_{Pruning_enabled_}$SNTR | 0.3572 | 16 | 0.19017 | 0.047543 | |

**Table 20**
Paired-samples *t* test (experimental data vs best M5 model tree for test dataset 2). Sig. (0.875) is the *p* value which is greater than 0.05 indicating that there is no significant difference between the means of the two data sets.

| Pair | Paired differences | | | | | *t* | *df* | Sig. (2-tailed) |
|---|---|---|---|---|---|---|---|---|
| | | | | 95% Confidence interval of the difference | | | | |
| | Mean | Std. Deviation | Std. Error mean | Lower | Upper | | | |
| Experimental_SNTR and $M5_{Pruning_enabled_}$SNTR | 0.0026 | 0.065797 | 0.016449 | −0.03244 | 0.03769 | 0.16 | 15 | 0.875 |

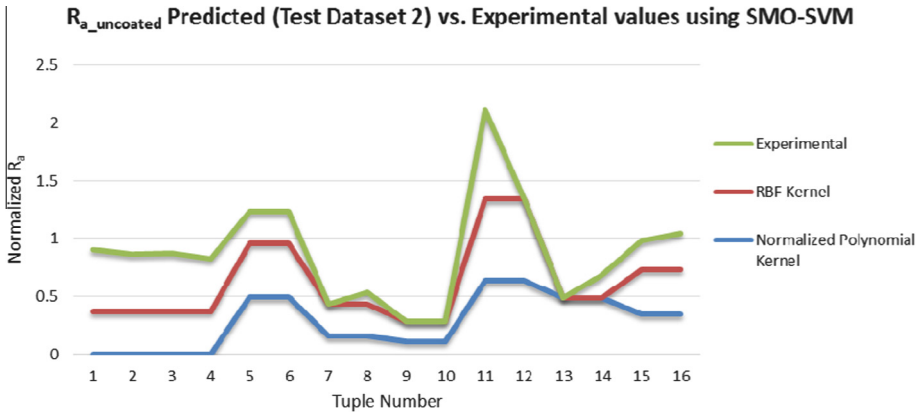


**Fig. 13.** Test dataset 2 (uncoated tool), predicted $R_a$ by SMO-SVM vs. experimental data.

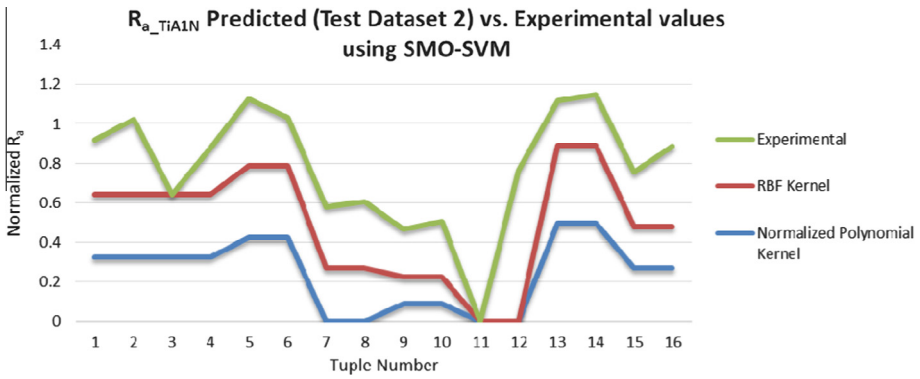


**Fig. 14.** Test dataset 2 (TiA1N tool), predicted $R_a$ by SMO-SVM vs. experimental data.

and $p = 0.318$. Thus, the two means of experimental SNTR tool and the best SMO-SVM model are not significantly different from each other.

Considering the predicted $R_a$ value, the model trees and SMO-SVM based models can be evaluated as follows:

### *4.1. Experimental data vs. model trees (test dataset 1)*

For the experimental data, the minimum value of $R_a$ is 0.190 μm for SNTR tool (cf. Table 1). However, in case of model trees, the minimum predicted value of $R_a$ is 0.364 μm for test dataset 1 (cf. Section 3.3). Thus, the model tree has provided $R_a$ value which is 0.174 μm greater than the experimental data. This poor result can be attributed to the inappropriately

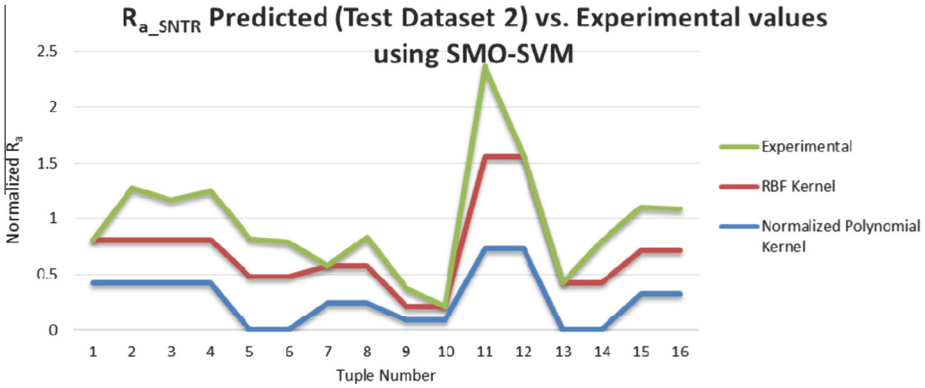


**Fig. 15.** Test dataset 2 (SNTR tool), predicted $R_a$ by SMO-SVM vs. experimental data.

**Table 21**
Statistics and correlations (experimental data vs. best SMO-SVM model for test dataset 2).

| | Mean | *N* | Std. Deviation | Std. Error mean | Correlation |
|---|---|---|---|---|---|
| Experimental_SNTR | 0.3895 | 16 | 0.20067 | 0.050168 | 0.769 |
| SMO-SVM-NormalizedPoly_SNTR | 0.4246 | 16 | 0.199629 | 0.049907 | |

**Table 22**
Paired-samples *t* test (experimental data vs best SMO-SVM model for test dataset 2). Sig. (0.318) is the p value which is greater than 0.05 indicating that there is no significant difference between the means of the two data sets.

| Pair | Paired differences | | | | | *t* | *df* | Sig. (2-tailed) |
|---|---|---|---|---|---|---|---|---|
| | | | | 95% Confidence interval of the difference | | | | |
| | Mean | Std. Deviation | Std. Error mean | Lower | Upper | | | |
| Experimental_SNTR and SMO-SVM-NormalizedPoly_SNTR | −0.035 | 0.136036 | 0.034009 | −0.10761 | 0.03736 | −1.033 | 15 | 0.318 |

designed training and test datasets as proven by the results for the test dataset 2 where carefully crafted training and test datasets resulted in the model tree providing better results.

### *4.2. Experimental data vs. SMO-SVM based model (test dataset 1)*

The minimum predicted value of $R_a$ by SMO-SVM is 0.128 μm for test dataset 1 (cf. Section 3.4). Comparing with the experimental data, it is evident that SMO-SVM based model has resulted in a minimum $R_a$ value which is 0.062 μm less than the experimental data. Although a good result, the paired samples *t*-test indicate that the difference in the means of the predicted and experimental datasets is significantly large thus implying that the generated model does not adequately captures the dynamics of the experimental data.

### *4.3. Experimental data vs. model trees (test dataset 2)*

For the experimental data, the minimum value of $R_a$ is 0.190 μm for SNTR tool (cf. Table 1). However, in case of model trees, the minimum predicted value of $R_a$ is 0.182 μm for test dataset 2 (cf. Section 3.6). Thus, the model tree has provided $R_a$ value which is 0.008 μm less than the experimental data. This result is better than the test dataset 1 where the model tree predicted an $R_a$ value which was 0.174 μm greater than the experimental data. Since the data tuples and the algorithm used to generate the model are the same as training and test dataset 1, the better results can safely be attributed to the carefully redesigned training and test datasets 2.

### *4.4. Experimental data vs. SMO-SVM based model (test dataset 2)*

The minimum predicted value of $R_a$ by SMO-SVM is 0.186 μm for test dataset 2 (cf. Section 3.7). Comparing with the experimental data, it is evident that SMO-SVM based model has resulted in a minimum $R_a$ value which is 0.004 μm less than the experimental data. This result is better than the test dataset 1 where, although the predicted $R_a$ value was less than the experimental data but the difference in the means of the predicted and experimental datasets was significantly large

**Table 23**
Minimum value of surface roughness. Model tree and SMO-SVM have been used for the first time in this research and also used the empirical data with different training and test sets (training and test dataset 2) than the ANN model of [7].

| Technique | Minimum value of $R_a$ (μm) | Percentage change in $R_a$ value (compared with lowest experimental value) | Remarks |
|---|---|---|---|
| Experimental | 0.190 | 0 | Actually observed minimum $R_a$ value. A properly validated predicated value lower than this is desirable |
| Model tress (test dataset 1) | 0.364 | +91.5 | Poorly designed training and test datasets resulting in a predicted value too high than the experimental $R_a$ value |
| SMO-SVM (test dataset 1) | 0.128 | −32.6 | Poorly designed training and test datasets resulting in significant difference between the means of predicated and experimental datasets indicating a poor model |
| Model tree (test dataset 2) | 0.182 | −4.2 | Carefully redesigned training and test datasets resulting in a lower $R_a$ value by a good model with no significant difference between the means of the predicated and experimental datasets |
| SMO-SVM (test dataset 2) | 0.186 | −2.1 | Carefully redesigned training and test datasets resulting in a lower $R_a$ value by a good model with no significant difference between the means of the predicated and experimental datasets |
| Regression [7] | 0.187 | −1.5 | Model tree and SMO-SVM (test dataset 2) predicted even further lower $R_a$ value |
| ANN [7] | 0.188 | −1.05 | Model tree and SMO-SVM (test dataset 2) predicted even further lower $R_a$ value |

indicating that the SMO-SVM model does not bear good resemblance with the experimental data. However, for test dataset 2 there was no significant difference in the means of the predicted and experimental datasets implying that the SMO-SVM has now adequately captured the process dynamics.

### *4.5. Model tree vs. SMO-SVM*

For test dataset 1, both model tree and SMO-SVM did not perform well with model tree predicting a very high $R_a$ value than the experimental data and SMO-SVM model failing on the paired sample *t*-test due to significantly large difference in the means of the experimental and predicted datasets. However, with a carefully redesigned test dataset 2, both the model tree and SMO-SVM performed well by predicting an $R_a$ value lower than the experimental value, having line patterns of predicted data bearing good similarity with the experimental data and having no significant difference in the means of the predicted data and the experimental data. As the minimum predicted $R_a$ value is 0.182 μm and 0.0186 μm for model tree and SMO-SVM based model respectively, it is evident that the model tree has given $R_a$ value which is 0.004 μm less than SMO-SVM based model.

## 5. Conclusion

Two techniques, model trees and SMO-SVM were used for the first time in this research to build regression models for predicting the minimum value of surface roughness $R_a$ in the end milling process. Table 23 summarizes the minimum value of $R_a$ for the experimental data, model trees and SMO-SVM. Also the results reported in [7] using regression and ANN techniques are included for comparison. It is evident from Table 23 that both the techniques, model trees and SMO-SVM reported in this paper (when applied on properly crafted test dataset 2) have performed better than the regression and ANN techniques reported in [7] as they have reduced the minimum $R_a$ value of experimental data by 4.2 and 2.1% respectively.

This work also signifies the importance of careful partitioning of data into training and test datasets for obtaining more accurate models. As demonstrated through detailed experimentation, both the techniques did not perform well when applied to the training and test sets used in a previous study [7] (referred to as training and test dataset 1 in this paper). In [7] the last 16 rows of Table 2 were taken as the training data. Many of these rows have different output values corresponding to identical input attribute values that can lead to poorer training and degraded performance of the resulting regression models. However, when we repartitioned the data by taking the first eight rows of Table 2 as the training dataset that contain all unique attribute values and the remaining 16 rows as the test dataset, it resulted in better training of both, model trees and SMO-SVM models as demonstrated through better results (in terms of lower predicted $R_a$ value) that were appropriately validated through observing the plots of predicted data vs. the experimental data and the difference in the means of the predicted and experimental datasets.

The reliability of the results obtained through these regression models can be enhanced further by increasing the amount empirical data through additional experimentation. In future, we will try to collect additional surface roughness related experimental data and apply the techniques of model trees and SMO-SVM to it, to further test their modeling accuracy.

For the domain of surface roughness modeling, this discovery of the application of these two techniques (model trees and SMO-SVM) along with the guidelines for their proper use through carefully designed training and test datasets is a step

forward which will help the practitioners in this field in building more accurate models of surface roughness for other cutting tools and different experimental settings.

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